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Belur Math, Howrah - 711202

M. Sc ADMISSION TEST - 2022

MATHEMATICS

Date : 20/08/2022 **Full Marks : 50** Time : 12 noon - 1:00 pm

Instructions for the candidates

- Answer all questions.
- Each question has 4 options out of which only one is correct.
- Tick (✓) the correct option on OMR SHEET.
- The tick (✓) must be very clear – if it is smudgy or not clear, no marks will be awarded.
- Each correct answer carries **2 marks** and for each incorrect answer **1 mark** will be deducted.
- Unanswered questions will not be awarded.
- Multiple answers will be considered as wrong answer.
- Calculator is **not** allowed.
- All the notations have their usual significance.

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1. Consider the vector space V of polynomials in one-variable upto degree 2, over $\mathbb{R}$. Let W_1, W_2 be two subspaces of V, each of dimension 2. Then, which of the following statements is true?
(a) $W_1 \cup W_2 = V$.
(b) $W_1 \cap W_2$ is a subspace of dimension greater than equal to 1.
(c) $W_1 - (W_1 \cap W_2)$ will be a subspace of dimension 1.
(d) $V - (W_1 \cup W_2)$ is the trivial subspace of V.

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by $T(e_1) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$; $T(e_2) = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$; $T(e_3) = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ where $\{e_1, e_2, e_3\}$ is the standard basis of $\mathbb{R}^3$. Then, which of the following matrix gives the correct matrix representation of the linear transformation T with respect to the standard basis?

(a) $\begin{pmatrix} 1 & 3 & 2 \\ 2 & 2 & 1 \\ 3 & 1 & 3 \end{pmatrix}$ **(b)** $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 3 & 2 \\ 2 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ **(d)** $\begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

3. If $T : \mathbb{R}^3 \to \mathbb{R}^3$, takes $T(1, 2, 3) = (1, 0, 0)$; $T(3, 2, 0) = (0, 1, 0)$ and $T(3, 0, 0) = (0, 1, 2)$. Then, which of the following statements is incorrect?
(a) T is invertible.
(b) T is one-one, but not onto.
(c) $ker(T)$ is trivial.
(d) $Rank(T) \geq 2$.

4. Let A be a 3×3 real matrix. Which of the following polynomials definitely can not annihilate A?
(a) $x^2 + 2x + 1$.
(b) $x^2 + x + 1$.
(c) $x^3 + 3x^2 + 3x + 1$.
(d) $x^3 - 3x^2 + 3x - 1$.

5. Let A be an infinite, closed, bounded subset of a metric space (X, d). Then
(a) A is compact.
(b) A has a limit point.
(c) Every sequence in A has a convergent subsequence.
(d) None of (a), (b) and (c).

6. Let d be the usual metric on $\mathbb{R}$. Consider the map $f : (\mathbb{R}, d) \to (\mathbb{R}, d)$ given by $f(x) = x + x^3, \forall x \in \mathbb{R}$. Which one of the following is false?
(a) f is continuous.
(b) f maps every closed set to a closed set.
(c) f maps every open set to an open set.
(d) f preserves distance.

7. Consider the subset $A = \{n + \frac{1}{n} : n \in \mathbb{N}\}$ of $\mathbb{R}$ in usual metric. Then
(a) A has exactly one limit point.
(b) A has countably infinite number of limit points.
(c) A has uncountably many limit points.
(d) A has no limit point.

8. In the group $\mathbb{Z}$, the subgroup generated by $\{9, 10\}$ is
(a) $\mathbb{Z}$ **(b)** $9\mathbb{Z}$ **(c)** $10\mathbb{Z}$ **(d)** $90\mathbb{Z}$.

9. The order of the permutation $(17)(56)(13)(45)(18)(52)$ is
(a) 2 **(b)** 4 **(c)** 8 **(d)**16.

10. Which of the following statement is not true in general
(a) Every ED is an UFD
(b) Every UFD is a PID
(c) Every ED is a PID
(d) Every UFD is an Integral Domain.

11. If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle ϕ with the positive direction of x-axis, then its equation is
(a) $y \sin\phi - x\cos\phi = a\cos 2\phi$, **(b)** $y\cos\phi - x\sin\phi = a\cos 2\phi$,
(c) $y\cos\phi + x\sin\phi = a\sin 2\phi$, **(d)** None of (a), (b) and (c).

12. The radius of curvature at any point (r, θ) on the cardiode $r = a(1 - \cos\theta)$ is
(a) $\frac{4}{3}\sqrt{2ar}$, **(b)** $\frac{2}{3}\sqrt{\frac{2a}{r}}$, **(c)** $\frac{2}{3}\sqrt{2ar}$, **(d)** None of (a), (b) and (c).

13. The series $\displaystyle\sum_{n=1}^{\infty} \frac{x}{n(1+nx)^2}$ is
(a) convergent on [0,1] but not uniformly,
(b) divergent on [0,1],
(c) uniformly convergent on [0,1],
(d) None of (a), (b) and (c).

14. Let $f(x) = \begin{cases} 4, & x \in [0,2] \cap \mathbb{Q} \\ 5, & x \in [0,2] - \mathbb{Q} \end{cases}$, then

(a) $\overline{\int_0^2} f(x)dx = 10,\ \underline{\int_0^2} f(x)dx = 8,$ **(b)** $\overline{\int_0^2} f(x)dx = 5,\ \underline{\int_0^2} f(x)dx = 4,$

(c) $\overline{\int_0^2} f(x)dx = 10,\ \underline{\int_0^2} f(x)dx = 5,$ **(d)** None of (a), (b) and (c).

15. The particular integral for $(D^2 - 4D + 4)y = xe^{2x}$ where $D \equiv \frac{d}{dx}$, is

(a) $\frac{x^3e^{2x}}{6}$, **(b)** $\frac{x^3e^{2x}}{12}$, **(c)** $\frac{x^2e^{x}}{3}$, **(d)** None of (a), (b) and (c).

16. If $u_{xx} - (2\sin x)u_{xy} - (\cos^2 x)u_{yy} - (\cos x)u_y + 16 = 0$, then
(a) the partial differential equation is parabolic,
(b) the partial differential equation is hyperbolic,
(c) the partial differential equation is elliptic,
(d) None of (a), (b) and (c).

17. Let $f : \mathbb{C} \to \mathbb{C}$ be a function which is analytic at z_0, then
(a) f is not differentiable at z_0,
(b) $\exists\ m \in \mathbb{N}$ such that f is differentiable at z_0 upto m^{th} order,
(c) f is infinitely many times differentiable at z_0,
(d) None of (a), (b) and (c).

18. If $I = \int_C \frac{(\sin z)^2 e^{2z}}{(3z-8)^9} dz$ where C is the square with vertices at (0, 0), (1, 0), (1, 1) and (0, 1) in the positive sense, then
(a) $I = 2$ **(b)** $I = 1$ **(c)** $I = 0.5$ **(d)** $I = 0$.

19. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that $f(x,y) = \begin{cases} x\sin\frac{1}{y} + y\sin\frac{1}{x}, & xy \neq 0, \\ 0, & xy = 0. \end{cases}$ Then,
(a) Repeated limits of f exist at $(0,0)$, and they are equal
(b) Repeated limits of f exist at $(0,0)$, and they are unequal
(c) Repeated limits of f do not exist at $(0,0)$, but the double limit of f exist at $(0,0)$
(d) Neither repeated limits nor double limit of f exist at $(0,0)$.

20. Suppose you are using the method involving Lagrange multiplier to find the shortest distance between $(-1,4)$ and the line $12x - 5y + 71 = 0$. What are the values of the shortest distance and Lagrange multiplier, respectively?
(a) 3, $\frac{5}{13}$ **(b)** 3, 1 **(c)** 3, $\frac{6}{13}$ **(d)** 1, 1.

21. If the Trapezoidal rule with the single interval [0,1] is exact for approximating the integral $\int_0^1 (x^3 - cx^2)dx$, then the value of c is equal to

(a) 0.5 **(b)** 1 **(c)** 1.5 **(d)** 2

22. If $p(x) = 2-(x+1)+x(x+1)-bx(x+1)(x-a)$ interpolates the points (x,y) in the following table

x	-1	0	1	2
y	2	1	2	-7

then $a + b =$
(a) 0 **(b)** 1 **(c)** 3 **(d)** 4.

23. The locus of the poles of tangents to the parabola $y^2 = 4ax$ with respect to the parabola $y^2 = 4bx$ is
(a) $y^2 = \frac{4b^2}{a}x$ (b) $y^2 = \frac{4a^2}{b}x$ (c) $y^2 = \frac{4b}{a}x$ (d) $y^2 = \frac{4a}{b}x$.

24. The equation of the right circular cone which contains three positive co-ordinate axes is
(a) $xy + 2yz + zx = 0$
(b) $2xy + yz + zx = 0$
(c) $xy + yz + 2zx = 0$
(d) $xy + yz + zx = 0$.

25. The quadric represented by $26x^2 + 20y^2 + 10z^2 – 4yz – 16zx – 36xy + 52x – 36y – 16z + 25 = 0$ has
(a) a unique centre
(b) a line of centres
(c) a plane of centres
(d) no centre.

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