RAMAKRISHNA MISSION VIDYAMANDIRA

Belur Math, Howrah – 711 202

P.G. ADMISSION TEST – 2017
MATHEMATICS

Date : 05-07-2017 **Full Marks : 50** Time: 11·00 a.m. – 1·00 p.m

Instructions for the candidate

Write the answer key(s) in the appropriate space provided. Each question carries ***2 marks*** *for correct answer and* ***–1 mark*** *for wrong answer. A few questions are given with more than one correct answer keys, the candidate has to write each correct answer keys — otherwise it would be treated as a wrong answer.*

Name of the student : ______________________________

Application No. : ______________________________

Signature of the invigilator : ______________________________

1. Let A be a 3×3 matrix and $A^2 - A = 0$. Then

   a) A must be a null matrix b) A must be an identity matrix

   c) Rank of A is 0 or 1 d) A is diagonalizable

   Ans. ____________

2. The minimal polynomial of $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix}$ is

   a) $x - 2$ b) $(x-2)(x-5)$ c) $(x-2)^2(x-5)$ d) $(x-2)^3(x-5)$

   Ans. ____________

3. A is an upper triangular matrix with all diagonal entries zero. Then I + A must be

   a) Invertible b) Idempotent c) Singular d) Nilpotent

   Ans. ____________

4. Let f and g be two real-valued continuous functions defined on [0, 1] such that $\sup_{x\in[0,1]} f(x) = \sup_{x\in[0,1]} g(x)$. Then

   a) $\exists x \in [0,1]$ such that f(x) = g(x)

   b) $\exists x \in [0,1]$ such that f(x) = g(x) – 2

   c) $\exists x \in [0,1]$ such that f(x) = g(x) = $\sup_{t\in[0,1]} f(t)$

   d) $\exists x \in [0,1]$ such that $f(x)^2 + 2f(x) = g(x)^2 + 2g(x)$

   Ans. ____________

5. Let $f:[0,1]\to\mathbb{R}$ has IVP and for all $c\in\mathbb{R}$, $f^{-1}(c)$ is closed. Then (where IVP is intermediate value property)

a) f is continuous b) f is discontinuous c) f is differentiable d) none of these

Ans.

6. Let $\{x_n\}$ be a real sequence such that $x_1=1,\ x_2=2,\ x_{n+2}=\frac{x_{n+1}+1}{x_n}\ \forall\, n\in\mathbb{N}$. Then x_{122} equals

a) 1 b) 2 c) 3 d) none of these

Ans.

7. Let $\{f_n\}$ be a sequence of non-decreasing real-valued functions from [0,1] to [0,1]. If $\lim_{n\to\infty} f_n(x)=f(x)$ pointwise and f is continuous then

a) $f_n\to f$ uniformly b) $f_n\to f$ but not uniformly

c) $f_n(x)\le f(x)\ \forall\, x\in[0,1]$ d) none of these

Ans.

8. Let $f(x,y)=\frac{xy^2}{x^2+y^4},(x,y)\in\mathbb{R}^2$. Then

a) $\lim_{\substack{x\to0\\y\to0}} f(x,y)$ does not exist

b) $\lim_{\substack{x\to0\\y\to0}} f(x,y)$ exists

c) Both $\lim_{y\to0}\lim_{x\to0} f(x,y)$ and $\lim_{x\to0}\lim_{y\to0} f(x,y)$ exist and are equal

d) $\lim_{y\to0}\lim_{x\to0} f(x,y)=\lim_{x\to0}\lim_{y\to0} f(x,y)=\lim_{\substack{x\to0\\y\to0}} f(x,y)$

Ans.

9. Let (X,d) be a metric space such that each sequence in X has a Cauchy subsequence. Then

a) X is bounded b) X is complete

c) X is compact d) X has at least one limit point

Ans.

10. Let G be a noncommutative group of order 10. Then G has

a) No element of order 5 b) Only one element of order 5

c) 4 elements of order 5 d) 7 elements of order 5

Ans.

11. Let n be a positive integer with 24 positive divisors. Then n equals

a) 756 b) 3087 c) 3200 d) none of these

Ans.

12. Let A = {1, 2, 3}. The number of symmetric relations on A is

a) 2^4 b) 2^6 c) 9 d) none of these

Ans.

13. Let C[0, 1] denote the set of all real-valued continuous functions on [0, 1]. Define '+' and '·' on C[0, 1] as follows :

For $f, g \in C[0,1]$, $\left.\begin{array}{r}(f+g)(x) = f(x)+g(x)\\ (f \cdot g)(x) = f(x)g(x)\end{array}\right\}(x \in [0,1])$ Then

a) C[0, 1] is a commutative ring with identity but not an integral domain

b) C[0, 1] is an integral domain

c) C[0, 1] is a skew-field

d) If $A_{1/2} = \left\{ f \in C[0,1] : f\left(\frac{1}{2}\right) = 0 \right\}$ then $A_{1/2}$ is an ideal of C[0, 1]

Ans.

14. Let A, B, C be three events such that P(A) = 0·4, P(B) = 0·5, $P(A \cup B) = 0{\cdot}6$, P(C) = 0·6 and $P(A \cap B \cap C^C) = 0{\cdot}1$. Then $P(A \cap B \mid C)$ is equal to

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{1}{5}$

Ans.

15. In an examination there are 80 questions each having four choices. Exactly one of these four choices is correct and the other three are wrong. A student is awarded 1 mark for each correct answer and –0·25 for each wrong answer. If a student ticks the answer for each question randomly, then the expected value of his/her total marks in the examination is

a) –15 b) 0 c) 5 d) 20

Ans.

16. If $\vec{A} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, then the value of $\int_C \vec{A}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path C : x = t, y = t^2, z = t^3 is

a) 5 b) 4 c) –4 d) –5

Ans.

17. The missing term in the following table is :

x : 0	1	2	3	4
f(x) : 4	3	4	?	12

a) 5 b) 6 c) 7 d) 8

Ans.

18. Choose the correct answer or answers : [Here $\mathbb{R}^{\#}$ = {set of all positive real numbers including 0}]

a) The domain of Laplace transformation is the set of all bounded continuous functions defined in $\mathbb{R}^{\#}$

b) The domain of Laplace transformation is the set of all piecewise continuous functions defined in $\mathbb{R}^{\#}$

c) The domain of Laplace transformation is the set of all piecewise continuous functions defined in $\mathbb{R}^{\#}$ and which are of some exponential order as the variable (of the function) tends to infinity

d) none of these

Ans.

19. Solution of $(D^4+2D^3+3D^2+2D+1)y=0$, where $D \equiv \frac{d}{dx}$ is

a) $e^{-\left(\frac{x}{2}\right)}\left[(C_1+C_2x)\cos\left(\frac{\sqrt{3}x}{2}\right)+(C_3+C_4x)\sin\left(\frac{\sqrt{3}x}{2}\right)\right]$, where $C_i, i=1,2,3,4$ are arbitrary constants

b) $e^{-\left(\frac{x}{2}\right)}\left[(C_1+C_2x)\cos h\left(\frac{\sqrt{3}x}{2}\right)+(C_3+C_4x)\sin h\left(\frac{\sqrt{3}x}{2}\right)\right]$, where $C_i, i=1,2,3,4$ are arbitrary constants

c) $e^{-\left(\frac{x}{2}\right)}\left[(C_1+C_2x)\cos\left(\frac{\sqrt{3}x}{2}\right)+(C_3x+C_4x^2)\sin\left(\frac{\sqrt{3}x}{2}\right)\right]$, where $C_i, i=1,2,3,4$ are arbitrary constants

d) $e^{-\left(\frac{x}{2}\right)}\left[(C_1+C_2x)\cosh\left(\frac{\sqrt{3}x}{2}\right)+(C_3x+C_4x^2)\sinh\left(\frac{\sqrt{3}x}{2}\right)\right]$, where $C_i, i=1,2,3,4$ are arbitrary constants

Ans.

20. Choose the correct statement or statements :

a) The function $y=x^2+x+3$ is a solution of the initial value problem $x^2y''-2xy'+2y=6$ with y(0) = 3 and $y'(0)=1$ on the real line

b) The function $y=5x^2+x+3$ is a solution of the initial value problem $x^2y''-2xy'+2y=6$ with y(0) = 3 and $y'(0)=1$ on the real line

c) The function $y=99x^2+x+3$ is a solution of the initial value problem $x^2y''-2xy'+2y=6$ with y(0) = 3 and $y'(0)=1$ on the real line

d) The function $y=-x^2+x+3$ is a solution of the initial value problem $x^2y''-2xy'+2y=6$ with y(0) = 3 and $y'(0)=1$ on the real line

Ans.

21. Let $f(z)=\frac{1+z}{1-z}$, where $z\in\mathbb{C}\setminus\{1\}$ which of the following is/are true?

a) If $A=\{|z|<1\}$ and $B=\{\text{Re}(z)>0\}$, then $f(A)\subseteq B$, where $f(A)=\{f(a)\,|\,a\in A\}$

b) If $A=\{|z|>1, \text{Im}(z)>0\}$ and $B=\{\text{Re}(z)<0, \text{Im}(z)>0\}$, then $f(A)\subseteq B$, where $f(A)=\{f(a)\,|\,a\in A\}$

c) If $A=\{|z|>1\}$ and $B=\{\text{Im}(z)>0\}$, then $f(A)\subseteq B$, where $f(A)=\{f(a)\,|\,a\in A\}$

d) If $A=\{|z|<1\}$ and $B=\{\text{Re}(z)<0\}$, then $f(A)\subseteq B$, where $f(A)=\{f(a)\,|\,a\in A\}$

Ans.

22. A particle describes a curve whose equation is $r^2 = a^2 \sin 2\theta$ under a force to the pole. The force is proportional to

a) r^{-3} b) r^{-11} c) r^{-5} d) r^{-7}

Ans.

23. The lengths AB and AD of the sides of a rectangle ABCD are 2a and 2b. The inclination to AB of one of the principal axes at A is

a) $\frac{1}{2}\tan^{-1}\left\{\frac{3ab}{a^2-b^2}\right\}$ b) $\frac{1}{2}\tan^{-1}\left\{\frac{3ab}{2(a^2+b^2)}\right\}$ c) $\frac{1}{2}\tan^{-1}\left\{\frac{3ab}{2(a^2-b^2)}\right\}$ d) $\frac{1}{2}\tan^{-1}\left\{\frac{3ab}{4(a^2-b^2)}\right\}$

Ans.

24. A rough uniform board of mass m and length 2a, rests on a smooth horizontal plane and a man of mass M walks on it from one end to the other. The distance through the board moves in this time is

a) $\frac{Ma}{M+m}$ b) $\frac{2ma}{M+m}$ c) $\frac{2Ma}{M+m}$ d) $\frac{Ma}{2(M+m)}$

Ans.

25. A uniform cubical box of edge ‘a’ is placed on the top of a fixed sphere. The least radius of the sphere for which the equilibrium will be stable is

a) $\frac{a}{2}$ b) $\frac{2a}{3}$ c) a d) $\frac{a}{4}$

Ans.

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