

# RAMAKRISHNA MISSION VIDYAMANDIRA

Belur Math, Howrah – 711 202

## ADMISSION TEST – 2017 MATHEMATICS (Honours)

Date : 14-06-2017

Full Marks : 50

Time: 11·00 a.m. – 12·00 noon

### Instructions for the candidate

Answer all the questions given below. Each question carries **2 marks** for correct answer and **-1 mark** for wrong answer. Tick (✓) the correct option. The tick must be very clear — if it is smudgy or not clear, no marks will be awarded. **Calculator is not allowed.**

Name of the student : \_\_\_\_\_

Application No. : \_\_\_\_\_

Signature of the invigilator : \_\_\_\_\_

- Equation of the ellipse with x and y axes as its major and minor axes respectively, passing through the point (-3, 1) and with eccentricity  $\sqrt{\frac{2}{5}}$  is  
a)  $3x^2 + 5y^2 = 32$ ,      b)  $6x^2 + 10y^2 = 32$ ,      c)  $5x^2 + 3y^2 = 32$ ,      d) none of these.
- The locus of the point of intersection of the straight lines  $\frac{tx}{a} + \frac{y}{b} - t = 0$  &  $\frac{x}{a} - \frac{ty}{b} + 1 = 0$  is  
a) a circle,      b) a parabola,      c) an ellipse,      d) none of these.
- If one of the diameters of the circle A with equation  $x^2 + y^2 - 2x - 6y + 6 = 0$  is a chord to the circle B with centre (2, 1) then the radius of the circle B is  
a)  $\sqrt{3}$ ,      b)  $\sqrt{2}$ ,      c) 3,      d) 2.
- 99<sup>th</sup> term of the series  $2 + 7 + 14 + 23 + 34 + \dots$  is  
a) 9999,      b) 9998,      c) 10000,      d) none of these.
- The sum of all natural numbers between 0 and 100 which are not divisible by 5 is  
a) 4000,      b) 4050,      c) 1050,      d) 4005.
- If  $\vec{a}, \vec{b}, \vec{c}, \vec{d}$  are unit vectors such that  $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$  and  $\vec{a} \cdot \vec{c} = \frac{1}{2}$  then  
a)  $\vec{a}, \vec{b}, \vec{c}$  are non-coplanar,      b)  $\vec{b}, \vec{c}, \vec{d}$  are non-coplanar,  
c)  $\vec{b}, \vec{d}$  are non-parallel,      d)  $\vec{a}, \vec{d}$  are parallel and  $\vec{b}, \vec{c}$  are parallel.
- If  $I = \int_{-1}^2 |x \sin \pi x| dx$  then I equals  
a)  $\frac{1}{\pi}$ ,      b)  $\frac{2}{\pi}$ ,      c)  $\frac{4}{\pi}$ ,      d)  $\frac{5}{\pi}$ .

8. Let  $f$  be a non-negative function defined on the interval  $[0,1]$ . If  $\int_0^x \sqrt{1 - \{f'(t)\}^2} dt = \int_0^x f(t) dt$ ,  $0 \leq x \leq 1$  and  $f(0) = 0$  then
- a)  $f\left(\frac{1}{2}\right) < \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) > \frac{1}{3}$ ,      b)  $f\left(\frac{1}{2}\right) > \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) > \frac{1}{3}$ ,
- c)  $f\left(\frac{1}{2}\right) < \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) < \frac{1}{3}$ ,      d)  $f\left(\frac{1}{2}\right) > \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) < \frac{1}{3}$ .
9. If  $I = \int_{\frac{1}{e}}^e |\log x| \frac{dx}{x^2}$ , then  $I$  equals
- a)  $2\left(1 - \frac{1}{e}\right)$ ,      b) 2,      c)  $\frac{2}{e}$ ,      d) 0.
10. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 2x + \sin x$  for  $x \in \mathbb{R}$ . Then  $f$  is
- a) one-one and onto,      b) one-one but not onto,  
c) onto but not one-one,      d) neither one-one nor onto.
11. For the circle  $x^2 + y^2 = r^2$ , the value of  $r$  for which the area enclosed by the tangents drawn from the point  $(6,8)$  to the circle and the chord of contact is maximum is
- a) 10,      b) 5,      c)  $5\sqrt{2}$ ,      d) none of these.
12. Which of the following function is differentiable at  $x = 0$ ?
- a)  $\cos(|x|) + |x|$ ,      b)  $\cos(|x|) - |x|$ ,      c)  $\sin(|x|) + |x|$ ,      d)  $\sin(|x|) - |x|$ .
13. Let  $\mathbb{Q}, \mathbb{R}, \mathbb{R} - \mathbb{Q}$  denote respectively the set of all rational numbers, real numbers and irrational numbers. Suppose  $p \in \mathbb{R} - \mathbb{Q}$  and  $\mathbb{Q} + p = \{x + p : x \in \mathbb{Q}\}$ . Then
- a)  $\mathbb{Q} \cup (\mathbb{Q} + p) = \mathbb{R}$ ,      b)  $\mathbb{Q} + p = \mathbb{R} - \mathbb{Q}$  if  $p$  is transcendental,  
c)  $\mathbb{Q} + p = \mathbb{R} - \mathbb{Q}$  if  $p$  is algebraic,      d)  $\mathbb{Q} + p$  is a proper subset of  $\mathbb{R} - \mathbb{Q}$ .
14. Let  $|X| = 4$ ,  $|Y| = 3$ . The number of surjective maps from  $X$  to  $Y$  is
- a) 24,      b) 30,      c) 36,      d) none of these.
15. Let the relation  $\rho$  be defined on  $\mathbb{Z}$  (the set of all integers) by ' $a \rho b$  iff  $ab \geq 0$ ;  $a, b \in \mathbb{Z}$ '. Then  $\rho$  is
- a) Reflexive but neither symmetric nor transitive,      b) Reflexive and symmetric but not transitive,  
c) Reflexive and transitive but not symmetric,      d) An equivalence relation.
16. Let  $A$  be any  $2 \times 2$  matrix and  $B$  be its adjoint. Then the determinant of the product matrix  $AB$  is
- a) 1,      b)  $\det A$ ,      c)  $(\det A)^2$ ,      d) none of these.

17. Let A and B be two matrices of order  $2 \times 2$ . If they have a common row and same determinant, then which one of the following statements is true?

- a)  $A = B$ ,  
 b) adjoint of A = adjoint of B,  
 c)  $\det(AB) = (\det(A))^2$ ,  
 d)  $\det(A+B) = \det A + \det B$ .

18. The smallest number whose square exceeds the number by 2 is

- a) 1,                                      b) 2,                                      c) -1,                                      d) -2.

19. The sum of two prime numbers is 61. The sum of their squares is

- a) 2195,                                      b) 2875,                                      c) 3485,                                      d) none of these.

20. The determinant of the matrix  $A = \begin{bmatrix} x^2 + x & x^3 + x^2 & x^4 + x^3 \\ x^2 & x^3 & x^4 \\ x^2 - x & x^3 - x^2 & x^4 - x^3 \end{bmatrix}$ ,  $x \in \mathbb{R}$  is

- a) a polynomial in x of degree 7,                                      b) 0,  
 c) a polynomial in x of degree 9,                                      d) none of these.

21. If  $E_1$  and  $E_2$  are the events associated to a random experiment and if  $E_1$  implies  $E_2$ , then

- a)  $P(E_1) < P(E_2)$ ,                      b)  $P(E_1) \leq P(E_2)$ ,                      c)  $P(E_1) \geq P(E_2)$ ,                      d)  $P(E_1) > P(E_2)$ .

22. The probability of drawing a card which is either a spade or a king from a well-shuffled pack of cards is

- a)  $\frac{1}{26}$ ,                                      b)  $\frac{17}{52}$ ,                                      c)  $\frac{1}{52}$ ,                                      d)  $\frac{4}{13}$ .

23. The area of the region bounded by the curves  $y = x^2$  and  $x = y^2$  is

- a)  $\frac{1}{2}$ ,                                      b)  $\frac{1}{3}$ ,                                      c)  $\frac{1}{4}$ ,                                      d) none of these.

24. The curves  $x^2 - y^2 = 8$ ,  $xy = 3$  intersect at an angle

- a)  $\frac{\pi}{3}$ ,                                      b)  $\frac{\pi}{2}$ ,                                      c)  $\frac{\pi}{4}$ ,                                      d) none of these.

25. The maximum value of  $\left(\frac{1}{x}\right)^x$  is

- a)  $\left(\frac{1}{e}\right)^e$ ,                                      b)  $e^e$ ,                                      c)  $e^{\frac{1}{e}}$ ,                                      d) none of these.

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