RAMAKRISHNA MISSION VIDYAMANDIRA

Belur Math, Howrah – 711 202

P.G ADMISSION TEST – 2016
MATHEMATICS

Date : 24-06-2016 **Full Marks : 50** Time: 2 hours

Name of the student : ____

Application No. : ______________________________

Signature of the student : ____________________ Signature of the Invigilator : ________________

Instructions for the candidate

Answer all the questions given below. Each question carries **2 marks** for correct answer and **–1 marks** for wrong answer. Tick (✓) the correct option. The tick must be very clear — if it is smudgy or not clear, no marks will be awarded. **Calculators are allowed.**

---

1. Let G be a group of order 15. Then
   a) G is cyclic
   b) G is abelian but may not be cyclic
   c) G may not be abelian
   d) G contains only one element of order 3

2. Let G be a nonabelian group of order 39. Then the number of elements of order 3 in G is
   a) 1 b) 3 c) 26 d) 38

3. Let $\mathcal{A}$ denote the set of all order preserving bijections from $\mathbb{Z}$ to $\mathbb{Z}$ where $\mathbb{Z} = \{0, \pm1, \pm2, \pm3, ...\}$. Then $\mathcal{A}$ is
   a) a singleton set b) a finite set c) a countably infinite set d) an uncountable set

4. Consider the equivalence relation ρ on $\mathbb{R}$ defined by '$x \rho y$ iff $x - y \in \mathbb{Q}$; $x, y \in \mathbb{R}$' where $\mathbb{R}$ and $\mathbb{Q}$ respectively denote the set of all real numbers and the set of all rational numbers. Then
   a) Equivalence classes are dense in $\mathbb{R}$ but not all of them are countable
   b) All equivalence classes are countable and dense in $\mathbb{R}$
   c) Equivalence classes are countable but not all of them are dense in $\mathbb{R}$
   d) The only equivalence class which is dense in $\mathbb{R}$ is $\mathbb{Q}$

5. Suppose G is a nonabelian group of order 10 with centre Z(G). Then the order of Z(G) is
   a) 1 b) 2 c) 5 d) not possible to determine

6. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = |x|,\ x \in \mathbb{R}$ ($\mathbb{R}$ denotes the set of all real numbers). Suppose A is the set of all functions $g : \mathbb{R} \to \mathbb{R}$ such that g is continuous and $g^2 = f$. The number of elements in A is
   a) 0 b) 1 c) 2 d) 4
   [$g^2(x) = \{g(x)\}^2$ for all $x \in \mathbb{R}$]

7. Let $A \subseteq \mathbb{R}$ such that $A \cap A^d = \phi$ ($\mathbb{R}$ denotes the set of all real numbers, A^d denotes the derived set of A and ϕ denotes the empty set). Then

a) A^d must be empty
b) A^d is at most a singleton set
c) A^d is at most a finite set
d) A^d may be infinite

8. Consider the metrics 'd_1', 'd_2', 'd_3' defined on $\mathbb{N}$ (the set of all natural numbers) as follows :

$$d_1(x,y) = |x-y|,\ d_2(x,y) = \left|\frac{1}{x}-\frac{1}{y}\right|,\ d_3(x,y) = \begin{cases}1, & x \neq y\\ 0, & x = y\end{cases}$$

Then

a) No two of d_1, d_2, d_3 are equivalent
b) d_1 and d_3 are not equivalent
c) d_2 and d_3 are not equivalent
d) d_1, d_2, d_3 all are equivalent

9. The value of $\tau(180)$ is

a) 12
b) 13
c) 18
d) 19

[$\tau(n)$ denotes the number of positive divisors of n]

10. The number of zeros with which the decimal representation of 50! terminates is

a) 10
b) 11
c) 12
d) 13

11. The value of $\phi(360)$ is

a) 90
b) 96
c) 92
d) 98

[ϕ denotes the Euler's phi-function]

12. The value of $\lim\limits_{x\to 0} \dfrac{\int_0^{x^2} e^{\sqrt{1+t}}dt}{x^2}$ is

a) 1
b) 0
c) e^2
d) e

13. The value of $\lim\limits_{n\to\infty}\left[\dfrac{1}{n+1}+\dfrac{1}{n+2}+...+\dfrac{1}{n+n}\right]$ is

a) log 2
b) log (1+e)
c) log 3
d) $\log\dfrac{1}{2}$

14. Taking space length h = 0·5 the value of $\Delta^6\left[\left(1-\dfrac{x}{3}\right)\left(1-\dfrac{4}{5}x^2\right)\left(1-\dfrac{x^3}{3}\right)\right]$ is

a) 1
b) –1
c) 0
d) $\dfrac{\lfloor 6 \times 4}{45}$

15. Let f be a differentiable function such that $M_2 = \max\limits_{a\le x\le b}|f''(x)|$. Then the maximum error in linear interpolation on [a,b] is given by

a) $\dfrac{hM_2}{8}$
b) $\dfrac{h^2M_2}{8}$
c) $\dfrac{h^2M_2}{2}$
d) $\dfrac{h^2M_2}{6}$

16. One root of the equation $e^x - 3x^2 = 0$ lies in the interval (3, 4). Then the least number of iterations of the bisection method so that $|\text{error}| \le 10^{-3}$ is

a) 10 b) 8 c) 6 d) 4

17. In an examination 30% of the students failed in Mathematics, 15% of the students failed in English and 10% of the students failed in both Mathematics and English. A student is chosen at random. If he failed in English then the probability that he passed in Mathematics is

a) $\frac{1}{2}$ b) $\frac{1}{10}$ c) $\frac{1}{3}$ d) $\frac{7}{10}$

18. Let X be a continuous random variable with p.d.f. f(x) given by $f(x) = \begin{cases} \theta x + \frac{1}{2} & , \quad -1 < x < 1 \\ 0 & , \quad \text{elsewhere} \end{cases}$

where θ is a constant. Then the value of θ for which var(X) is maximum is

a) 1 b) 2 c) $\frac{1}{4}$ d) 0

19. Orthogonal trajectories of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is

a) $x^{4/3} - y^{4/3} = c^{4/3}$ b) $x^{4/3} + y^{4/3} = c^{4/3}$ c) $x^{2/3} - y^{2/3} = c^{2/3}$ d) $(xy)^{2/3} + 1 = c^{2/3}$

[a and c are constants]

20. If P be the point with co-ordinates (2, 3, –1), then the equation of the plane through P at right angle to the straight line OP, O is the origin, is

a) $x + 4y - 5z = 14$ b) $3x + 2y - z = 14$ c) $2x + 3y - z = 14$ d) $4x + y + 5z = 14$

21. The greatest and the least distance from the point (2, –1, 1) to the sphere $x^2 + y^2 + z^2 - 8x + 4y - 6z + 4 = 0$ is given by

a) g.d = 8, ℓ.d = 2 b) g.d = 4, ℓ.d = 1 c) g.d = 9, ℓ.d = 4 d) g.d = $\frac{8}{3}$, ℓ.d = $-\frac{2}{3}$

[g.d = greatest distance and ℓ.d = least distance]

22. Value of $\int_0^{\infty} \frac{e^{-3t} - e^{-6t}}{t} dt$ is

a) $\frac{1}{2}\log 2$ b) log2 c) $\frac{3}{2}\log 2$ d) none of these

23. The Laplace Transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$ is

a) $\sqrt{\frac{2\pi}{p}}\ e^{-\frac{1}{4p}}$ b) $\sqrt{\frac{\pi}{2p}}\ e^{-\frac{1}{2p}}$ c) $\sqrt{\frac{\pi}{p}}\ e^{-\frac{1}{2p}}$ d) $\sqrt{\frac{\pi}{p}}\ e^{-\frac{1}{4p}}$

24. A particle performing a S.H.M of period T about a centre O passes through a point P with a velocity v in the direction $\overrightarrow{OP}$. If OP is equal to x and the particle returns to P in time t then t is equal to

a) $\frac{\pi}{T}\tan^{-1}\frac{2\pi x}{vT}$ b) $\frac{T}{\pi}\tan^{-1}\frac{2\pi x}{vT}$ c) $\frac{\pi}{T}\tan^{-1}\frac{vT}{2\pi x}$ d) none of these

25. A rough uniform board of mass m and length 2a rests on a smooth horizontal plane and a man of mass M walks on it from one end to the other. The distance through the board moves in this time is

a) $\frac{2am}{M+m}$ b) $2a\left(1+\frac{m}{M}\right)$ c) $\frac{2aM}{M+m}$ d) none of these

———— × ————