RAMAKRISHNA MISSION VIDYAMANDIRA

Belur Math, Howrah – 711 202

M.Sc ADMISSION TEST - 2015

MATHEMATICS

Date : 17-07-2015 **Full Marks : 50** Time : 11·00 am – 01·00 pm

Answer as many questions as you can. Maximum you can score is 50.
Use separate Answer books for each group.

Group – A

1. Let G be a non commutative group of order 10. Show that Z(G), the centre of G contains only the identity element. [5]
2. Let G be a group of order 8 and x be an element of G of order 4. Prove that $x^2 \in Z(G)$, the centre of G. [5]

Group – B

3. Find two linear operators T and U on $\mathbb{R}^2$ such that $TU = 0$ but $UT \neq 0$. [5]
4. Let V be an n-dimensional vector space over a field F, and let $B = \{\alpha_1, ..., \alpha_n\}$ be an ordered basis for V. Let $T : V \to V$ be a linear operator defined by $T\alpha_i = \alpha_{i+1}, i = 1, ..., n-1$, $T\alpha_n = 0$. What is the matrix A of T with respect to the ordered basis B of V? [5]

Group – C

5. i) Find all $A \subseteq \mathbb{R}$ such that $\bar{A} = A^\circ$. [3]

   ii) Give an example of a set $A \subseteq \mathbb{R}$ such that $\overline{A^\circ} \neq A$. [2]
6. If every subsequence of a sequence in $\mathbb{R}$ has a further subsequence converging to 1, show that the original sequence converges to 1. [5]
7. A function $f : [0,1] \to \mathbb{R}$ is continuous on [0,1] and $\int_0^x f(t)\,dt = \int_x^1 f(t)\,dt$ for all $x \in [0,1]$.

   Prove that $f \equiv 0$. [5]

Group – D

8. a) Suppose an urn contains 10 white and 15 black balls. Let a ball be drawn at random from the urn and kept aside without studying the result. A second ball is drawn from the urn at random. What is the probability that the second ball drawn is white? [3]

   b) What does the equation $x^2 = 1$ represent geometrically? Explain your answer logically. [2]
9. a) Compare the volume of a tetrahedron with that of the tetrahedron formed by the centroids of its faces. [3]

   b) Show that $\Delta \ln f(x) = \ln\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$. [2]

Group – E

10. Find the singular solution of the differential equation

$$\sin\left(x\frac{dy}{dx}\right)\cos y = \cos\left(x\frac{dy}{dx}\right)\sin y + \frac{dy}{dx}.$$ [5]

11. Solve the partial differential equation :

$$y^2p - xyq = x(z - 2y)$$

by Lagrange's method, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. [5]

Group – F

12. A solid homogeneous cone of height h and semi-vertical angle α oscillates about a diameter of its base. Find the length of the simple equivalent pendulum. [5]

13. Using Laplace Transform, solve the differential equation

$$(D^2 + m^2)y = a\cos nt$$

given $y = Dy = 0$ when t = 0 and $D \equiv \frac{d}{dt}$. [5]

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