## *Semester II*

### Paper II [100 Marks]

### Module I (50)

### Algebra IB (25)

### [Classical Algebra]

1. Weierstrass' inequality, Cauchy - Schwarz inequality, Arithmetic, geometric, harmonic means and weighted means and related theorems.

2. Complex numbers - Definition and field structure, Conjugate of a complex number, modulus of a complex number, Triangle inequality, Amplitude of a complex number, De Moivre's theorem, roots of a complex number, nth roots of unity, exponential and logarithm of a complex number. Definition of $a^z\left(a \neq 0\right)$

3. Polynomials and polynomial equations, Addition and multiplication of polynomials, Division algorithm, Remainder theorem and factor theorem, Fundamental theorem of classical algebra (statement only) and its consequences. Polynomials with real co efficient and related theorems, Rolles theorem (statement only) and its consequences, Multiple roots and related theorems, Descartes rule of signs ( statement only) and its consequences, relation between roots and coefficients, Transformation of equation, Reciprocal equation, Binomial equation, Special roots and related theorems, Cardan's method for solving cubic equations, Ferrari's method for solving biquadratic equations.

**References**

[1] Higher Algebra (Classical) – S. K. Mapa.
[2] Higher Algebra – Barnard & Child.

### Analysis IB (25)

1. Infinite Series of real numbers:
a) Convergence, Cauchy's criterion of convergence.
b) Series of non-negative real numbers: Tests of convergence – Cauchy's condensation test. Comparison test, Kummer's test. Statements and applications of : Abel – Pringsheim's Test, Ratio Test , Root test, Raabe's test, Bertrand's test, Logarithmic test and Gauss's test.
c) Series of arbitrary terms : Absolute and conditional convergence
d) Alternating series : Leibnitz test.
e) Non-absolute convergence : Abel's and Dirichlet's test (statements and applicatins). Riemann's rearrangement theorem and rearrangement of absolutely convergent series.

2. Point set in one dimension: Open cover of a set. Compact set in R, Heine- Borel Theorem.

3. Continuity of a function at a point. Continuity of a function on an interval and at an isolated point. Algebra of continuous functions. Continuity of composite functions. Continuous function on a closed and bounded interval and its properties. Discontinuity of function, type of discontinuity. Step function. Piecewise continuity. Continuity of monotone

function. Uniform continuity, Lipschitz condition and uniform continuity. Related theorems on uniform continuity.

4. Derivatives of real valued functions of a real variable: Darboux theorem, Rolle's theorem, Mean value theorems of Lagrange and Cauchy. Taylor’s theorem on closed and bounded interval with Lagrange’s and Cauchy’s form of remainder. Maclaurin’s theorem as a consequence of Taylor’s theorem. Statement of Maclaurin’s Theorem on infinite series expansion. Expansion of $e^x$, log(1+x), $(1+x)^m$, sin x, cos x with their range of validity. Statement of L Hospital’s rule and its consequences. Point of local extremum of a function in an interval. Sufficient condition for the existence of a local extremum of a function at a point. Problems on local maxima - minima.

**References**

[1] Principles of Mathematical Analysis – Walter Rudin.
[2] Mathematical Analysis - T. M. Apostol.
[3] Real analysis – Goldberg.
[3] Real Analysis - S. K. Mapa.

## Module II (50)

### Linear Algebra I (20)

1. Matrices of real and complex numbers : Algebra of matrices. Symmetric and skew-symmetric matrices. Hermitian and skew-Hermitian matrices. Orthogonal matrices.

2. Determinants: Definition, Basic properties of determinants, Minors and cofactors. Laplaces method. Vandermonde’s determinant. Symmetric and skewsymmetric determinants. (No proof of theorems). Adjoint of a square matrix. Invertible matrix, Non-singular matrix. Inverse of an orthogonal Matrix.

3. Elementary operations on matrices. Echelon matrix. Rank of a matrix. Determination of rank of a matrix (relevant results are to be state only). Normal forms. Elementary matrices. Statements and application of results on elementary matrices. Systems of linear equations.

4. Vector space: Definitions and examples, Subspace, Union and intersection of subspaces. Linear sum of two subspaces. Linear combination, independence and dependence. Linear span. Generators of vector space. Dimension of a vector space. Finite dimensional vector space. Examples of infinite dimensional vector spaces. Replacement Theorem, Extension theorem. Extraction of basis.

**References**

[1] Linear Algebra – Hoffman and Kunze.
[2] Linear Algebra – Bhimasankaram and Rao.
[3] Linear Algebra A Geometrical Approach – S. Kumaresan.
[4] University Algebra – N. S. Gopalakrishnan.
[5] Higher Algebra – S. K. Mapa.
[6] Linear Algebra – Friedberg, Insel and Spence

## Optimization Technique (30)

1**.** Definition of L.P.P. Formation of L.P.P. from daily life involving inequations. Graphical solution of L.P.P. Basic solutions and Basic Feasible Solution (BFS) with reference to L.P.P. Matrix formulation of L.P.P. Degenerate and Non-degenerate B.F.S.

2**.** Hyperplane, Convex set, Cone, extreme points, convex hull and convex polyhedron. Supporting and Separating hyperplane. The collection of all feasible solutions of an L.P.P. constitutes a convex set. The extreme points of the convex set of feasible solutions correspond to its B.F.S. and conversely. The objective function has its optimal value at an extreme point of the convex polyhedron generated by the set of feasible solutions.
(the convex polyhedron may also be unbounded). In the absence of degeneracy, if the L.P.P. admits of an optimal solution then at least one B.F.S. must be optimal. Reduction of a F.S. to a B.F.S.

3**.** Slack and surplus variables. Standard from of L.P.P. theory of simplex method. Feasibility and optimality conditions.

4**.** The algorithm. Two phase method. Degeneracy in L.P.P. and its resolution.

5**.** Duality theory: The dual of the dual is the primal. Relation between the objective values of dual and the primal problems. Relation between their optimal values. Complementary slackness, Duality and simplex method and their applications.

6**.** Transportation and Assignment problems. Mathematical justification for optimality criterion. Hungarian method. Traveling Salesman problem.

**References**

[1] Linear Programming : Method and Application – S. I. Gass.
[2] Linear Programming – G. Hadley.
[3] An Introduction to Linear Programming & Theory of Games – S. Vajda.
[4] An Introduction to L. P. P. and Game – Ghosh and Chakravarty.
[5] Linear Programming- P. M. Karak.