CLASSICAL MECHANICS
Full marks: 50
(60 Hours)

Degree of freedom, Generalized Coordinates, Unilateral and bilateral constraints, Holonomic and non-holonomic system, Scleronomic and Rheonomic systems, Principle of virtual work, D'Alembert's principle, Lagrange's equations of first and second kind (holonomic and non-holonomic system), Euler-Lagrange's equation, velocity-dependent potentials, Rayleigh's dissipation function, Cyclic or ignorable coordinates, Energy conservation in Lagrangian formulation.

Hamilton's principle, Necessary and sufficient conditions of Hamilton's principle, Legendre transformation, Hamilton's canonical equation of motion, Derivation of Hamilton's equation of motion from Lagrange's equations and Hamilton's principle, Derivation of Lagrange's equation from Hamilton's principle, Extension of Hamilton's principle to systems with constraints, Routhian, Principle of least action, Noether's Theorem, Brachitochrone for uniform force field, Hamilton's Principle Function, Hamilton's Characteristic Function, Lagrange's and Poisson brackets, Invariance of Lagrange's and Poisson brackets, integral invariants of Poincare, Canonical transformations, Differential forms and generating functions, Liouville's theorem, Jacobi's Identity, Poisson's Theorem, Jacobi-Poisson Theorem.

Hamilton-Jacobi equation, Solution to the time dependent Hamilton-Jacobi equation, Conditions for Separability of Coordinates, Solution of one dimensional simple harmonic oscillator problem by Hamilton-Jacobi method, Action Angle Variables, Adiabatic Invariance.

Principle of linear momentum, Principle of angular momentum, Euler's dynamical equations, Eulerian angles, Motion of a symmetrical top, Theory of small oscillations.

References

1. Classical Mechanics - N. C. Rana and P. S. Joag, TATA-McGraw Hill Publishing Company Ltd.
2. Classical Mechanics - H. Goldstein, C. P. Poole and J. Safko, Pearson
3. Analytical Mechanics - Louis N. Hand and Janet D. Finch
4. Principles of Mechanics - Synge and Griffith
5. Classical Dynamics - D. Greenwood, Prentice Hall of India,
6. A Text Book of Dynamic- F. Chorlton
7. Mathematical Methods of Classical Mechanics- V. I. Arnold
8. Classical Mechanics- Alexei Deriglazov
9. Classical Mechanics- Dieter Strauch
10. Theoretical Mechanics- N. G. Chetaev

ORDINARY DIFFERENTIAL EQUATIONS
Full Marks - 50
(60 hours)

1. Preliminaries – Initial Value problem and the equivalent integral equation, mth order equation in d-dimensions as a first order system, concepts of local existence, existence in the large and uniqueness of solutions with examples.

2. Basic Theorems – Ascoli-Arzela Theorem. A Theorem on convergence of solutions of a family of initial-value problems.

3. Picard-Lindelof Theorem – Peano's existence Theorem and corollary. Maximal intervals of existence. Extension Theorem and corollaries. Kamke's convergence Theorem. Kneser's Theorem (Statement only).

4. Differential inequalities and Uniqueness – Gronwall's inequality. Maximal and minimal solutions. Differential inequalities. A Theorem of Winter. Uniqueness Theorems. Nagumo's and Osgood's criteria.

5. Egres pointstand Lyapunov functions. Successive approximations.

6. Variation of constants, reduction to smaller systems. Basic inequalities, constant coefficients. Floquet Theory. Adjoint systems, Higher order equations.

7. Linear second order equations – Preliminaries. Basic facts. Theorems of Sturm. Sturm Liouvilee Boundary value Problems.

References

1. P. Hartman, Ordinary Differential Equations, John Wiley (1964).
2. E.A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, McGraw-Hill, NY (1955).
3. G.F. Simmons : Differential Equaitons.

FUNCTIONAL ANALYSIS
Full Marks - 50
(60 Hours)

1. Review of (i) Complete Metric Spaces. (ii) Completion of a Metric Space. (iii) Baire category theorem and applications. (iv) Banach Fixed Point Theorem.

2. Normed linear spaces, Banach spaces and examples. Quotient space of normed linear spaces and its completeness, equivalent norms, Riesz lemma, basic properties of finite dimensional normed linear spaces and compactness. Bounded linear transformations, normed linear spaces of bounded linear transformations, Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces, consequences of Hahn Banach theorem, dual spaces with examples. Open mapping and Closed Graph theorems, Uniform Boundedness Principle and some of its consequences, reflexive spaces.

3. Inner product spaces. Hilbert spaces, Orthonormal sets, Bessel's inequality, complete orthonormal sets and Parseval's identity, Structure of Hilbert spaces. Projection theorem, Riesz representation theorem, Reflexivity of Hilbert spaces.

4. Adjoint of an operator on a Hilbert space, Commutativity of operators on Hilbert spaces, Self-adjoint, positive, projection, normal and unitary operators.

References

l. G. Bachman and L. Narici. Functional Analysis. Academic Press, 1966.

2. N. Dunford and J.T. Schwartz, Linear Operators, Part-I. Interscience, New York, 1958.

3. C. Goffman and G. Pedric, First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987.

4. P.K.Jain, O.P.Ahuja and Khalil Ahmad, Functional Analysis, New Age International (P) Ltd. & Wiley Eastern Ltd., New Delhi, 1997.

5. B.V.Limaye, Functional Analysis, Wiley Eastern Ltd.

6. G.F.Simmons, Introduction to Topology and Modern Analysis. McGraw-Hill Book Company, New York. 1963.

7. A.E.Taylor. Introduction to Functional Analysis. John Wiley and Sons. New York, 1958.

8. J.B.Conway, A Course in Functional Analysis, Springer-Verlag, New York, 1990.

9. Walter Rudin, Functional Analysis, Tata McGraw-Hill Publishing Company Ltd.,

10. B.K. Lahiri: Elements of Functional Analysis.

NUMERICAL ANALYSIS II
Full marks: 50 (60 Hours)

(Theory - 20 marks)

1. Integration: Trapezoidal, Simpson's 1/3 rule,Weddle's rule, Gauss-Legendre quadrature, Method of undetermined parameters. Richardson's extrapolation technique and Romberg integration formula.

2. Least square polynomial approximation.

3. Numerical solution of ODE: Initial value problems: Taylor's series method, Picard method, Euler's method, Modified Euler's method, Outlines of Runge- Kutta 4^{th} order method, Milne's method, Adams- Bashforth method, Adams-Multan method.

4. Numerical Solution of PDE: Solution of Laplace's equation by Gauss-Seidal method, Iterative methods for the solution of one dimensional heat equation, Wave equation (outline).

5. Linear difference equation

(Practical - 30 marks)

The following problems should be done on computer using **C / MATLAB language**:

1. Integration: Trapezoidal, Simpson's 1/3 rule,Weddle's rule, Gauss-Legendre quadrature, Romberg integration formula.

2. Matrix eigen value problems: Jacobi method for symmetric matrices, Householder's method for symmetric matrices, Rutishauser method for arbitrary matrices, Power method.

1. Problems of curve fitting: $y = a + bx$, $y = a + bx+cx^2$ by Least square method.

2. Numerical solution of first order ODE: Euler method, Modified Euler method, 4^{th} order Runge-Kutta method, Milne's method, Adams-Multan method.

3. Numerical Solution of PDE: Solution of Laplace's equation by Gauss-Seidal method, Iterative methods for the solution of one dimensional heat equation, Wave equation.

4. Linear difference equation.

References

1. F. B. Hilderbrand: Introduction to Numerical Analysis, Mcgraw-Hill, 1974.
2. J.B. Scarborough: Numerical Mathematical Analysis, Johns Hopkins Press, 1966.
3. A. Ralston: A First Course in Numerical Analysis, Mcgraw-Hill, 1985.
4. J. Butcher: The Numerical Analysis for Ordinary Differential Equations, Wiley, 2008.
5. K. E. Atkinson: An Introduction to Numerical Analysis.
6. S. S. Sastry: Introductory Methods of Numerical Analysis.
7. M. K. Jain, S. R. K. Iyengar and R. K. Jain: Numerical Methods for Scientific and Engineering Computation.
8. Richard L. Burden, J. Douglas Faires and Annette M. Burden : Numerical Analysis.
9. W. H. Press, S. A. Teukolsky, W. T. Vetterling and Brian P. Flannery : Numerical Recipes in C.
10. E. Balaguruswamy : Numerical methods.