RAMAKRISHNA MISSION VIDYAMANDIRA

Belur Math, Howrah - 711202

M. Sc ADMISSION TEST - 2023

MATHEMATICS

Date : 18/08/2023 **Full Marks : 50** Time : 12 noon - 1:00 pm

Instructions for the candidates

- Answer all questions.
- Each question has 4 options out of which only one is correct.
- Tick (✓) the correct option on OMR SHEET.
- The tick (✓) must be very clear – if it is smudgy or not clear, no marks will be awarded.
- Each correct answer carries **2 marks** and for each incorrect answer **1 mark** will be deducted.
- Unanswered questions will not be awarded.
- Multiple answers will be considered as wrong answer.
- Calculator is **not** allowed.

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1. For the curve $y(x) = \begin{cases} \sqrt{1+x^2}\sin\frac{1}{x}, & when\ x \neq 0 \\ 0, & when\ x = 0 \end{cases}$,
   (a) $y = \pm d$ are asymptotes, where $d \in \mathbb{Q}^c$.
   (b) $y = \pm 1$ are the only asymptotes.
   (c) $x = \pm d$ are asymptotes, $\forall d \in \mathbb{R}$.
   (d) None of (a), (b) and (c) is true.

2. If C denotes the line joining from the point $z = i$ to $z = 1$ and let $I = \int_C \frac{dz}{z^4}$, then which of the following is not true
   (a) $|I| \leq 4\sqrt{2}$ **(b)** $|I| \leq 8$ **(c)** $|I| > 0$ **(d)** $|I| > 8$

3. The function $f(z) = x^2 + iy^2$ (where $z = x + iy$) is
   (a) analytic everywhere **(b)** analytic at some points of $\mathbb{C}$
   (c) nowhere analytic **(d)** differentiable everywhere.

4. The general solution of $\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = 5z + tan(y - 3x)$, (where z is a function of x and y) is
   (a) $5x - ln(5z - tan(y-3x)) = f(y-3x)$. **(b)** $5y - ln(5z + tan(y-3x)) = f(y-3x)$.
   (c) $5x - ln(5z + tan(y-3x)) = f(y-3x)$. **(d)** $5y - ln(5z - tan(y-3x)) = f(y-3x)$.
   (Here f is an arbitrary function)

5. The dimension of the subspace $W = \{(x, y, z, w) \in \mathbb{R}^4 : x = 2y, z = 3w\}$ in $\mathbb{R}^4$ is
   (a) 1 **(b)**2 **(c)** 3 **(d)**4.

6. If $\{(1, 2, 1), x, (3, 1, 1)\}$ is a basis for $\mathbb{R}^3$, then x can be
   (a)(4, 3, 2) **(b)**(−2, 1, 0) **(c)**(1, −3, −1) **(d)**(1, 2, 3).

7. The dimension of the subspace generated by $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 3 & 0 & 4 \end{bmatrix}$ in $M_{3\times 3}(\mathbb{R})$ is

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4.

8. In $\mathbb{Z}[\sqrt{-5}]$

(a) 3 is prime **(b)** 2 is prime **(c)** 6 has unique factorization **(d)** 3 is irreducible.

9. Let $\alpha \in \mathbb{R}$. If αx is the polynomial which interpolates the function $f(x) = \sin \pi x$ on $[-1, 1]$ at all zeros of the polynomial $4x^3 - x$, then $|\alpha| =$

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

10. While solving the equation $x^3 - 3x + 1 = 0$ using Newton-Raphson method with the initial guess of a root as 2, the value of the approximated root after one iteration is

(a) 3 **(b)** $\frac{5}{3}$ **(c)** $\frac{3}{5}$ **(d)** 5

11. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^4 + y^2} = ?$

(a) 0 **(b)** limit doesn't exist **(c)** 1 **(d)** 3

12. The maximum value of $f(x, y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$ is

(a) 30 **(b)** 39 **(c)** 40 **(d)** 49

13. Let $s_n = 1 + \frac{1}{\sqrt{2}} \cdots + \frac{1}{\sqrt{n}}$. Then, which of the following statement is true?

(a) $s_n > \sqrt{n}, \forall n \geq 3, n \in \mathbb{N}$.

(b) $s_n \leq \sqrt{n}, \forall n \in \mathbb{N}$.

(c) $\{s_n\}$ is a convergent sequence.

(d) There exists $N_0 \in \mathbb{N}$ such that $s_n > n$, for all $n \geq N_0$.

14. Which of the following statements is true for a continuous function $f : D \to \mathbb{R}$ where $D \subseteq \mathbb{R}$ is an interval.

(a) If D is a closed interval, then f has to be a bounded function.

(b) Let $x_n \in D, \forall n \in \mathbb{N}$. If $x_n \to x_0 \in \mathbb{R}$, then $\{f(x_n)\}$ must converge to a finite limit.

(c) f can be uniquely determined if f is known for all rational points in D.

(d) Intermediate value property may not hold if D is an open interval.

15. Let $f_n(x) = (1 - x)^n, x \in [0, 1], n \in \mathbb{N}$. Which of the following statements is true for the sequence $\{f_n\}$?

(a) The sequence $\{f_n\}$ converges uniformly on $[0, 1]$.

(b) The sequence $\{f_n\}$ converges uniformly on $[0, 1)$.

(c) The sequence $\{f_n\}$ converges uniformly on $(0, 1]$.

(d) None of the statements (a), (b), (c) is correct.

16. If $\int\limits_0^\infty \frac{\sin mx}{x} dx = \frac{\pi}{2}$ for $m > 0$, then $\int\limits_0^\infty \frac{\sin^3 x}{x} dx =$

(a) $\frac{\pi}{4}$ **(b)** $\frac{3\pi}{8}$ **(c)** $\frac{\pi}{2}$ **(d)** $\frac{3\pi}{4}$.

17. Suppose $A = \{n + \frac{1}{n} : n \in \mathbb{N}\}$. The number of limit points of A is
(a) 0 **(b)** 1 **(c)** 3 **(d)** ∞.

18. Let $\mathbb{Q} + \sqrt{2} = \{r + \sqrt{2} : r \in \mathbb{Q}\}$. The set $\mathbb{Q} + \sqrt{2}$ is
(a) closed in $\mathbb{R}$ **(b)** open in $\mathbb{R}$ **(c)** dense in $\mathbb{R}$ **(d)** bounded in $\mathbb{R}$.

19. Suppose $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is defined by
$$d(x, y) = \begin{cases} 1, & x \neq y \\ 0, & x = y \end{cases}$$
The metric space $(\mathbb{R}, d)$ is
(a) bounded but not totally bounded **(b)** totally bounded **(c)** separable **(d)** compact.

20. Suppose $d : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ is defined by $d(x, y) = \left|\frac{1}{x} - \frac{1}{y}\right|$. The metric space $(\mathbb{N}, d)$ is
(a) complete and separable **(b)** not complete
(c) separable but not bounded **(d)** bounded but not separable.

21. A subset A of a metric space is compact iff A is
(a) closed and bounded **(b)** complete
(c) totally bounded **(d)** complete and totally bounded

22. The polar equation of the ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$, if the pole be at its right-hand focus and the positive direction of the x-axis be the positive direction of the polar axis is
(a) $\frac{5}{r} = 3 + 2\cos\theta$ **(b)** $\frac{10}{r} = 3 + 2\cos\theta$ **(c)** $\frac{5}{r} = 3 - 2\cos\theta$ **(d)** $\frac{10}{r} = 3 - 2\cos\theta$

23. The equation of the right circular cone which contains three positive coordinate axes is
(a) $xy - yz - zx = 0$ **(b)** $xy + yz - zx = 0$
(c) $xy - yz + zx = 0$ **(d)** none of (a), (b) and (c).

24. A particle is projected with velocity u from a fixed point O on a straight line and moves along the line so that its acceleration f is given by $k\sqrt{t}$, where t is the time from the instant it leaves O and k is a constant. The distance from O in terms of acceleration is
(a) $\frac{uf^2}{k^2} + \frac{4f^5}{15k^4}$ **(b)** $\frac{uf^2}{k^2} - \frac{4f^5}{15k^4}$ **(c)** $\frac{uf^2}{k^2} + \frac{4f^5}{5k^4}$ **(d)** $\frac{uf^2}{k^2} - \frac{4f^5}{5k^4}$

25. The law of force, when a particle describes the curve $p^2 = ar$ under a force to the pole is proportional to
(a) r **(b)** $\frac{1}{r}$ **(c)** $\frac{1}{r^2}$ **(d)** $\frac{1}{r^3}$.

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