## Instructions for the candidate

Answer all the questions given below. Calculator is not allowed.

## Answer all the questions :

1. If $f(n)$ and $g(n)$ be two asymptotic non-negative functions, then prove that, $\max ((f(n)$, $\mathrm{g}(\mathrm{n}))=\theta(\mathrm{f}(\mathrm{n})+\mathrm{g}(\mathrm{n}))$. All symbols have their usual meaning. max means maximum.
2. The graph shown below has 8 edges with distinct integer edge weights. The Minimum Spanning Tree (MST) of this graph is of weight 36 and contains the edges: $\{(\mathrm{A}, \mathrm{C}),(\mathrm{B}, \mathrm{C}),(\mathrm{B}, \mathrm{E}),(\mathrm{E}, \mathrm{F})$, ( $\mathrm{D}, \mathrm{F}$ ) $\}$. The edge weights of only those edges which are in the MST are given in the figure shown below.


Find out the minimum possible sum of weights of all 8 edges of this graph with proper explanation.
3. A direct mapped cache memory of 1 MB has a block size of 256 bytes. The cache has an access time of 3 ns and a hit rate of $94 \%$. During a cache miss, it takes 20 ns to bring the first word of a block from the main memory, while each subsequent word takes 5 ns . The word size is 64 bits. Calculate average memory access time.
4. What is the minimum number of 2 -input NOR gates required to implement the following 4variable function expressed in Sum-of-Minterms form as $\mathrm{f}=\Sigma(0,2,5,7,8,10,13,15)$ ? Assume that all the inputs and their complements are available. Show the logic circuit also.
5. In a puzzle, there three pegs are mounted on a board together with $n$ disks of different sizes. Initially, these disks are placed on the first peg in order of size, with the largest on the bottom. Suppose our goal is to transfer all n disks from peg 1 to peg 3, where we cannot move a disk directly between pegs 1 and 3 . Each move of a disk must be a move involving peg 2 with the restriction that a disk never be placed on top of a smaller disk.
a) Find a recurrence relation for the number of moves required to solve the puzzle for n disks.
b) Find the explicit solution of this recurrence relation. Do not use the iteration method.
6. Do you think $\mathrm{K}_{5}$ is a bipartite graph? Does it form an Euler graph? Explain the reason behind your answer.
7. An urn contains two type-A coins and two type-B coins and one unbiased coin. When a type-A coin is flipped, it comes up heads with a probability $1 / 4$, and in case of type-B coin that is $3 / 4$. A coin is randomly chosen from the urn and flipped. Given that the flip landed with the tail face, what is the probability that it was a type A coin? Every coin must be landed with either head or tail face.
8. An organization is granted the block 16.0.0.0/8. The administrator wants to create 500 fixed-length subnets. Find the first and last address of subnet 500.
9. Julia says that Sarah and Fred say the truth. Sarah says that Peter is lying. Peter says that Fred says
the truth. Fred says that Peter lies or Julia lies. David says he lies. Write the above statements in propositional logic and through resolution find out who is/are lying.
10. Create an AVL Tree by inserting the values (show all rotations): 43, 69, 36, 5, 72, 26, 79, 59.
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