RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under the University of Calcutta)

Belur Math, Howrah - 711202

ADMISSION TEST - 2019

M.Sc. in Mathematics

Date : 27/06/2019 **Full Marks : 50** Time : 11:00 am - 1:00 pm

Instruction to the candidates

- The question set has been divided in two sections. Answer all the questions from each section.
- Section 1 contains multiple choice questions, each carrying **2 marks** for correct answer and **0 marks** for wrong answer. There may be more than one correct options. **Tick (✓)** the correct option(s) on the **OMR SHEET**. The tick must be very clear — if it is smudgy or not clear, no marks will be awarded. Answer will be treated correct only if all correct options are chosen.
- Section 2 contains subjective questions, each of **4 marks**. Answer this section in a separate sheet provided.
- Illegible answers will be considered as wrong answers.

Section 1 : Multiple Choice Questions

1. The co-ordinates of two points are (1,-2) and $(3\sqrt{3}+1, 1)$. The origin is shifted to the point (1,-2) and the new x-axis is the line joining the given points. Then the formula for the rigid motion is

   (a) $x = \frac{\sqrt{3}}{2}x' - \frac{1}{2}y' + 1$ and $y = \frac{1}{2}x' + \frac{\sqrt{3}}{2}y' - 2$

   (b) $x = \frac{1}{2}x' - \frac{\sqrt{3}}{2}y' + 1$ and $y = \frac{\sqrt{3}}{2}x' + \frac{1}{2}y' - 2$

   (c) $x = \frac{\sqrt{3}}{2}x' - \frac{1}{2}y' - 1$ and $y = \frac{1}{2}x' + \frac{\sqrt{3}}{2}y' + 2$

   (d) none of these.

2. The distance of the point (3,8,2) from the line

$$\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-2}{3},$$

   measured parallel to the plane $3x + 2y - 2z + 5 = 0$ is

   (a) 6 units (b) 7 units (c) 8 units (d) none of these.

3. Two numbers are chosen at random between 0 and 2. Then, the probability that their sum is less than 2 but, the sum of their squares is greater than 2 is

   (a) $\frac{1}{2} + \frac{\pi}{8}$ (b) $\frac{\pi}{8} - \frac{1}{2}$ (c) $\frac{1}{2} - \frac{\pi}{8}$ (d) $1 - \frac{\pi}{8}$

4. A particle of mass m falls down a smooth cycloid under its own weight, starting from the cusp. When it arrives at the vertex, the pressure on the curve is

(a) $\frac{1}{2}mg$ (b) $\frac{3}{2}mg$ (c) $4mg$ (d) None of these.

5. The torque about the point $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ of a force represented by $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ acting through the point $2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ is

(a) $\hat{\imath} + \hat{\jmath} - \hat{k}$ (b) $-\hat{\imath} - \hat{\jmath} + \hat{k}$ (c) $-\hat{\imath} + \hat{\jmath} + \hat{k}$ (d) None of these.

6. Let $f(x, y) = x^2 \sin(1/y) + y^{30} \sin(1/x)$ and A, B, C denote the double limit and the repeated limits in the following way.

$$A := \lim_{(x,y)\to(0,0)} f(x, y); \qquad B := \lim_{x\to 0} \lim_{y\to 0} f(x, y); \qquad C := \lim_{y\to 0} \lim_{x\to 0} f(x, y).$$

Then, which of the following is/are true?

(a) A exists but, none of B and C exist.
(b) Both B and C exist, but A does not exist.
(c) None of A, B and C exists.
(d) Only A and C exist.

7. Let $f : [a, b] \to \mathbb{R}$ where $a, b \in \mathbb{R}$ and $a < b$. State which of the following is/are true.

(a) f has an antiderivative iff f is continuous on $[a, b]$.
(b) If f is Riemann integrable on $[a, b]$, then there exists a function $\phi : [a, b] \to \mathbb{R}$ such that $\int_a^b f(x)dx = \phi(b) - \phi(a)$ and $\phi'(x) = f(x), \forall x \in [a, b]$.
(c) If f has an antiderivative, say ϕ, then f is Riemann integrable on $[a, b]$ and $\int_a^b f(x)dx = \phi(b) - \phi(a)$.
(d) If f is continuous on $[a, b]$, then it has infinitely many antiderivatives.

8. Let $I \subset \mathbb{R}$ be an open interval, let f: $I \to \mathbb{R}$ be differentiable on I, then at $a \in I$

$$f''(a) = \lim_{h\to 0} \frac{f(a + h) - 2f(a) + f(a - h)}{h^2}, \text{ if}$$

(a) $f''(a)$ exists.
(b) $f''(x)$ exists in a neighborhood of the point $x = a$ and is continuous at $x = a$.
(c) $f''(x)$ exists in a neighborhood of the point $x = a$.
(d) $f''(x)$ is continuous in a neighborhood of the point $x = a$.

9. If $a_1 = 2$, $a_{n+1} = a_n^2 - a_n + 1$ for $n > 1$ then

(a) $\sum_{i=1}^{\infty} \frac{1}{a_i}$=2 (b) $\sum_{i=1}^{\infty} \frac{1}{a_i}$=1 (c) $\sum_{i=1}^{\infty} \frac{1}{a_i}$=3 (d) $\sum_{i=1}^{\infty} \frac{1}{a_i}$ diverges

10. Let

$$a = \frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + ... + \frac{1}{3001}.$$

Then a is

(a) less than 1 (b) greater than 1 (c) less than $\frac{4}{3}$ (d) greater than $\frac{4}{3}$.

11. Which of the following is/are false?

(a) Any group of order 4 is abelian.
(b) Any group of order 7 is abelian.
(c) Any group of order 9 is abelian.
(d) Any group of order 24 is abelian.

12. Consider $\mathbb{N}$ with usual metric d. Which of the following is/are true?

(a) Any finite subset of $\mathbb{N}$ is an open ball in $(\mathbb{N}, d)$.
(b) The set $\{1, 2, ..., n\}$ is an open ball in $(\mathbb{N}, d)$ for all $n \in \mathbb{N}$.
(c) The set $\{n, n+1\}$ is an open ball in $(\mathbb{N}, d)$ for all $n \in \mathbb{N}$.
(d) Any infinite subset is an open ball in $(\mathbb{N}, d)$.

13. If f(x) is a continuous function on some interval $[a, b]$ and $f(a)f(b) < 0$ then the equation $f(x) = 0$ has

(a) at least one real root in the interval (a, b)
(b) an odd number of real roots in the interval (a, b)
(c) an even number of real roots in the interval (a, b)
(d) None of (a), (b) or (c).

14. If e^{2x} and e^{3x} are linearly independent solutions of $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$, then the solution satisfying the conditions $y(0) = 0$ and $y'(0) = 1$ is

(a) $c_1e^{2x} + c_2e^{3x}$ (b)$c_1e^{2x} - c_2e^{3x}$ (c)$e^{3x} - e^{2x}$ (d)$e^{3x} + e^{2x}$
where prime symbol($'$) denotes first order differentiation and c_1, c_2 are the arbitrary constants.

15. The order and the degree of the differential equation

$$\frac{d^2}{dx^2}\left(\frac{d^2y}{dx^2}\right)^{-\frac{3}{2}} = 0$$

are respectively

(a)1, 4 (b) 4, 1 (c)4, 4 (d)1, 1.

Section 2 : Subjective Questions

1. If F_1 and F_2 be two probability distribution functions of two random variables, then show that their convex combination $F = \lambda F_1 + (1 - \lambda)F_2$, $0 < \lambda < 1$ is also a probability distribution function of some random variable.

2. A solid homogeneous cone of height h and semi-vertical angle α oscillates about a diameter of its base. Find the length of the simple equivalent pendulum.

3. Let f(z)=$u+iv$ be an analytic function. Find f(z) (as a function of z) when $u-v = (x-y)(x^2+4xy+y^2)$.

4. Suppose G is a noncyclic group of order 39. Find the number of elements of G of order 3.

5. Let V be a finite dimensional inner product space and f be a linear functional on V. Prove that there exists a unique vector β in V such that $f(\alpha) = (\alpha|\beta)$ for all α in V.