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Belur Math, Howrah – 711 202

POST GRADUATE ADMISSION TEST – 2018

MATHEMATICS

Date : 30-06-2018 **Full Marks : 50** Time : 12.00 noon – 02.00 p.m

Instructions for the candidate

Answer all the questions. Each question carries **2 marks** for correct answer and **–1 mark** for wrong answer. Tick (✓) the correct option on the **OMR SHEET**. The tick must be very clear — if it is smudgy or not clear, no marks will be awarded.

1. Let I be an open interval in $\mathbb{R}$ and $f : I \to \mathbb{R}$ be a differentiable function. If $a \in I$, then state which of the following is/are true?

   a) $f''(a)$ exists iff $\lim\limits_{h\to 0}\dfrac{f(a+h)-2f(a)+f(a-h)}{h^2}$ exists and they are equal

   b) If $f''(a)$ exists then $\lim\limits_{h\to 0}\dfrac{f(a+h)-2f(a)+f(a-h)}{h^2}$ exists and they are equal

   c) If $\lim\limits_{h\to 0}\dfrac{f(a+h)-2f(a)+f(a-h)}{h^2}$ exists then $f''(a)$ exists and they are equal

   d) Existence of $f''(a)$ and $\lim\limits_{h\to 0}\dfrac{f(a+h)-2f(a)+f(a-h)}{h^2}$ are independent of one another.

2. If $\sum\limits_{n=1}^{\infty} a_n$ and $\sum\limits_{n=1}^{\infty} b_n$ be two series with real entries such that $|a_n| \le b_n, \forall n \in \mathbb{N}$, then which of the following is/are not true

   a) If $\sum\limits_{n=1}^{\infty} b_n$ converges, then $\sum\limits_{n=1}^{\infty} a_n$ also converges

   b) $\sum\limits_{n=1}^{\infty} a_n$ may be divergent even if $\sum\limits_{n=1}^{\infty} b_n$ is convergent

   c) If b_n's are bounded above, then $\sum\limits_{n=1}^{\infty} a_n$ is convergent

   d) If the set $\left\{\sum\limits_{n=1}^{m} b_n : m \in \mathbb{N}\right\}$ is bounded above, then $\sum\limits_{n=1}^{\infty} a_n$ is convergent

3. Let $f : (a,b) \to \mathbb{R}$ be a function such that $\lim\limits_{x\to c^-} f(x) \le \lim\limits_{x\to c^+} f(x),\ \forall\ c \in (a,b)$. Then which of the following is/are definitely true for f

   a) f is strictly monotonically increasing on (a,b)

   b) f is monotonically increasing on (a,b)

   c) f is continuous on (a,b)

   d) none of these is definitely true

4. Let $\{f_n(x)\}_{n=1}^{\infty}$ be a sequence of functions defined on an interval $[a,b] \subseteq \mathbb{R}$. If $f_n(x) \to f(x)$ uniformly on [a,b], then which of the following are true

a) If for each $n \in \mathbb{N}$, f_n is continuous on [a,b], then f is also continuous on [a,b]

b) If each f_n is Riemann integrable on [a,b], then f is also Riemann integrable on [a,b]

c) If f_n is differentiable on [a,b] for each $n \in \mathbb{N}$, with one sided derivatives, as applicable, at end points on [a,b], then f is also differentiable on [a,b]

d) If $\exists n_0 \in \mathbb{N}$ such that each f_n is continuous $\forall n \geq n_0$, then f is bounded on [a,b]

5. Let I be a compact interval in $\mathbb{R}$ and $f : I \to \mathbb{R}$ be a twice differentiable function such that $\exists p \in I^o$ for which $f(p) = 0$ but $f'(p) \neq 0$. Then, which of the following is true

a) $\exists \delta > 0$ such that f(x) changes its sign on $(p-\delta, p+\delta) \subseteq I$

b) $\exists \delta > 0$ such that for any $p_0 \in (p-\delta, p+\delta)$, the sequence $p_n = p_{n-1} - \dfrac{f(p_{n-1})}{f'(p_{n-1})}$, $n \in \mathbb{N}$ converges to p

c) $\exists \delta > 0$ such that $f'(x) \neq 0 \ \forall x \in (p-\delta, p+\delta)$

d) $f(x) = 0$ has a unique solution in I

6. Let 'd' be the usual metric on $\mathbb{N} = \{1, 2, 3, ...\}$. Then

a) Every subset of $\mathbb{N}$ is an open ball

b) For any n, the set $\{1, 2, 3, ..., n\}$ is an open ball

c) A finite set with even number of elements is not an open ball

d) If $A \subseteq \mathbb{N}$ is an open ball; $x, y \in A$, $x < z < y$ where $z \in \mathbb{N}$ then $z \in A$

7. Consider the set $\mathbb{R}$ of all real numbers and let 'd' be defined by $d(x,y) = \begin{cases} 1, & x \neq y \\ 0, & x = y \end{cases}$, then in $(\mathbb{R}, d)$.

a) Only finite subsets are compact

b) A subset A is compact if and only if A is closed and bounded

c) Any set is bounded

d) The set $\mathbb{N}$ is compact

8. If one of the eigen values of $A_{n\times n}$ is zero, then

a) the solution to the system of equations AX = C is unique

b) $\det(A) = 0$

c) $AX = 0$ has only trivial solution

d) $\det(A) \neq 0$

9. Let W be a vector space over $\mathbb{R}$ and let $T : \mathbb{R}^6 \to W$ be a linear transformation such that $S = \{Te_2, Te_4, Te_6\}$ spans $W(\{e_1, e_2, e_3, e_4, e_5, e_6\}$ is the standard basis for $\mathbb{R}^6)$. Then

a) S is a basis of W

b) $T(\mathbb{R}^6) \neq W$

c) $\{Te_1, Te_3, Te_4\}$ spans W

d) Ker (T) contains more than one element

10. The roots of the equation $z^n = (1+z)^n$

a) lie on a circle of radius 1
b) lie on a regular polygon of n sides
c) lie on a regular polygon of 2n sides
d) lie on a straight line

11. The function $f(z) = |z|^2$ is

a) continuous in the whole complex plane
b) differentiable at the origin
c) differentiable in the whole complex plane
d) analytic at the origin

12. Let $f(z) = u + iv$ is a non-constant analytic function. Then

a) v is harmonic conjugate of u
b) u is harmonic conjugate of v
c) if $\exists$ another function W which is harmonic conjugate of u then v – W is constant
d) u and v are harmonic functions

13. Find out which one of the following relations ρ are equivalence relation on S

a) $S = \mathbb{Z}$, $a \rho b$ iff $a^2 - b^2$ is a multiple of 11
b) $S = \mathbb{Z}$, $a \rho b$ iff $b = a^r$ for some $r \in \mathbb{N}$
c) $S = \mathbb{Z} \times \mathbb{Z}$, $(a,b) \rho (c,d)$ iff $a + d = b + c$
d) $S = \mathbb{Z} \times \mathbb{Z} \setminus \{(0,0)\}$, $(a,b) \rho (c,d)$ iff $ad = bc$

14. The number of idempotent (multiplicative) elements in $(\mathbb{Z}, +, \bullet)$ is

a) 0
b) 1
c) 2
d) 3

15. The equation of the surface passing through the parabolas z = 0, y^2 = 4ax and z = 1, y^2 = –4ax and satisfying the differential equation xr + 2p = 0, where $r = \frac{\partial^2 z}{\partial x^2}$, $p = \frac{\partial z}{\partial x}$; is

a) $z = -\frac{y^2}{8ax} + \frac{1}{2}$
b) $z = -\frac{y^2}{8ax} - \frac{1}{2}$
c) $z = -\frac{y^2}{8ax} + \frac{1}{4}$
d) none of these

16. If Laplace transform of F(t) is f(s) then the Laplace transform of F(at), 'a' is a constant, is

a) $\frac{1}{a} f\left(\frac{s}{a}\right)$
b) f(as)
c) $sf\left(\frac{1}{a}\right)$
d) none of these

17. The circulation of $\vec{F}$ round the curve C where $\vec{F} = (2x - y + 4z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z^3)\hat{k}$ and C is the circle $x^2 + y^2 = 9$, z = 0; is equal to

a) 9π
b) 18π
c) 2π
d) 0

18. A force $\vec{F}$ of magnitude 10 units acts along the line $\frac{x-2}{5} = \frac{y-1}{4} = \frac{z-3}{3}$. The moment of the force $\vec{F}$ about z-axis is

a) $\sqrt{2}$ unit
b) $2\sqrt{2}$ unit
c) $3\sqrt{2}$ unit
d) none of these

19. A rectangular plate swings in a vertical plane about one of its corners. If its period is one second then the length of the diagonal is

a) $\frac{3g}{2\pi^2}$ b) $\frac{3g}{4\pi^2}$ c) $\frac{3g}{8\pi^2}$ d) none of these

20. If h, h' be the greatest heights in two paths of a particle with a given velocity for a given range R, then

a) $\frac{R}{2} = \sqrt{hh'}$ b) $\frac{R^2}{16} = hh'$ c) $\frac{R^2}{8} = hh'$ d) $R^2 = hh'$

21. Forces 5P, 3P, 2P act along the sides OA, AB and BO of the equilateral triangle OAB of side 2a. Taking the axis OX along OA, and the axis OY perpendicular to it; The equation of the line of action of the resultant force is

a) $\sqrt{3}x - 5y = 6\sqrt{3}a$ b) $\sqrt{3}x + 5y = 6\sqrt{3}a$ c) $\sqrt{3}x - 5y = 3\sqrt{3}a$ d) $\sqrt{3}x + 5y = 3\sqrt{3}a$

22. In a Poisson distribution $P(X=1) = 2P(X=3)$. Then the variance of the distribution is

a) $\sqrt{2}$ b) $\sqrt{3}$ c) $2\sqrt{3}$ d) $3\sqrt{2}$

23. If X and Y are uncorrelated, then $\text{var}(aX + bY)$ is equal to

a) $a\,\text{var}(X) + b\,\text{var}(Y)$ b) $a^2\,\text{var}(X) + b^2\,\text{var}(Y)$

c) $a\,\text{var}(X) + b\,\text{var}(Y) + ab$ d) $a^2\,\text{var}(X) + b^2\,\text{var}(Y) + 2ab$

24. Let the origin be shifted to the point (1,1) and the axes be turned through an angle $\theta = \tan^{-1}\left(\frac{5}{12}\right)$. Then coordinates of the point which remain the same w.r.t both the system of coordinates are

a) (–2, 2) b) (2, 3) c) (–2, 3) d) (–2, –3)

25. The equation $\frac{3}{y-z} + \frac{4}{z-x} + \frac{5}{x-y} = 0$ represents

a) a plane b) a cone c) a cylinder d) pair of planes

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