RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2019

FIRST YEAR (BATCH 2018-21)

MATH FOR INDUSTRIAL CHEMISTRY (General)

Date : 24-05-2019

Time : 11.00 am – 2.00 pm

Paper : II

Full Marks : 75

[Use a separate Answer book for each group]

Group-A

Answer any three questions from questions no 1 to 5 [3×5]

1. a) Find a unit vector perpendicular to both the vectors $\vec{\alpha} = 2\hat{i} - 3\hat{j}$ and $\vec{\beta} = -\hat{i} + 2\hat{j} - 3k$. 3

   b) Find the value of 'a' such that the vectors $\vec{\alpha} = a\hat{i} + 2\hat{j} - 3k$ and $\vec{\beta} = \hat{i} - 4k$ are perpendicular. 2

2. Given three vectors $\vec{u}, \vec{v}, \vec{w}$ with $V = \vec{u}\cdot\left(\vec{v}\times\vec{w}\right) \neq 0$. Define three new vectors $\vec{a} = \frac{\left(\vec{v}\times\vec{w}\right)}{V}$, $\vec{b} = \frac{\left(\vec{w}\times\vec{u}\right)}{V}$, $\vec{c} = \frac{\left(\vec{u}\times\vec{v}\right)}{V}$. Verify that $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right) = \frac{1}{V}$. 5

3. If $\vec{a}, \vec{b}, \vec{c}$ be any three non-coplanar vectors then prove that, any vector $\vec{r}$ can be written as
$$\vec{r} = \frac{\left[\vec{r}\ \vec{b}\ \vec{c}\right]}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}\vec{a} + \frac{\left[\vec{a}\ \vec{r}\ \vec{c}\right]}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}\vec{b} + \frac{\left[\vec{a}\ \vec{b}\ \vec{r}\right]}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}\vec{c}\,.$$ 5

4. a) In a triangle ABC let $\vec{BC} = \vec{a}, \vec{CA} = \vec{b}$ and $\vec{AB} = \vec{c}$. Prove, by vector method, that $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ where a,b,c are the lengths of the sides BC,CA and AB respectively. 3

   b) Prove that $\left(\vec{b} - \frac{\vec{a}\,.\,\vec{b}}{\left|\vec{a}\right|^2}\vec{a}\right)$ is perpendicular to $\vec{a}$. 2

5. (a) A force of magnitude 5 units acts along the line joining the points A(3,-1,-6) and B(8,4,-1). Find the torque of the force about the point (1,2,-3). 3

   (b) Find, by vector method, the volume of the tetrahedron ABCD with vertices A(1,1,-1) , B(3,-2,-2) ,C(5,5,3) and D(4,3,2) 2

Answer any two from questions no 6 to 9 [2×5]

6. Show that $\int \frac{x^{n-1}}{f(x)}dx = \sum_{r=1}^{n} \frac{(a_r)^{n-1}}{f'(a_r)}\log(x - a_r)$ where $f(x) = \prod_{r=1}^{n}(x - a_r)$ $(a_i \neq a_k \text{ if } i \neq k)$

7. Obtain the reduction formula for $\int \frac{\sin^m x}{\cos^n x}dx$, where m , n are positive integers and hence evaluate $\int \frac{\sin^3 x}{\cos^2 x}dx$. (3+2)

8. Find $\lim_{n\to\infty} \frac{1}{n}\left[1+\left(\frac{2^2}{n^2}+1\right)+\left(\frac{4^2}{n^2}+1\right)+.......+\left(\frac{(2n-2)^2}{n^2}+1\right)\right]$

9. (a) Evaluate : $\int_0^{\frac{\pi}{4}} \log(1+\tan\theta)\,d\theta$. (3)

(b) State Fundamental theorem of integral calculus . (2)

Group-B

Answer any five questions from questions no 10 to 17 [5×5]

10. Use Cauchy's general principle of convergence to show that the sequence $\{u_n\}$, where $u_n = \sum_{i=1}^{n} \frac{1}{i}$ is not convergent .

11. Show that every convergent sequence of real numbers is bounded . Is the converse is true ? Justify your answer .

12. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\sin\frac{1}{n}}{\sqrt{n}}$.

13. Test that the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$.

14. State Rolle's Theorem . Verify the theorem for the function $f:[2,3]\to\mathbb{R}$ defined by $f(x)=x^2-5x+10$.

15. Evaluate : $\lim_{x\to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x\sin x}$.

16. Prove that the function $f(x,y)=x^2-2xy+y^2+x^4+y^4$ has a minimum at the origin.

17. a) Is it possible to evaluate $\lim_{x\to 0} \frac{\sin x}{x}$ using L' Hospital's Rule ? Justify .

b) Determine the maximum and minimum value of $f(x,y)=x^2+y^2-2x-2y$ subject to the constraint x+2y =4 using method of Lagrange multipliers .

Group-C

Answer any five questions from questions no 18 to 25 [5×5]

18. An elevator starts with 6 persons and stops at 8 floors of a building .What is the probability that no two person got down at the same floor ?

19. Let A, B be two events so that $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}, P(B/A)=\frac{1}{3}$ .Then

i) are A, B independent? Justify. 2

ii) are A, B mutually exclusive? Justify. 2

iii) find $P(A\cup B)$. 1

20. The probabilities of solving a problem by three students A, B, C are $\frac{3}{7}$, $\frac{3}{8}$, $\frac{1}{3}$ respectively. If all of them try independently, find the probability that the problem could be solved by one person only.

21. A seven digit number is formed by the digits 0 , 1 , 2 , 3 , 4 , 5 , 6 (without repetition) . Find the probability that the number formed is divisible by 4.

22. Find the 'mean' and 'variance' of Poission distribution.

23. The chance of a person hitting a target is $\frac{1}{3}$. How many times must be fire so that the probability of hitting the target at least once is more than 0.9.

24. 10 Fair coins are tossed simultaneously. Find the probability of getting

i) at least six heads . 2

ii) at most six heads . 3

25. In a normal distribution, 46% of the items are over 40 and 90% are under 75. Find the mean and standard deviation of the distribution. Given that, $\int_{-\infty}^{x}(2\pi)^{-\frac{1}{2}}e^{-\frac{t^2}{2}}dt = 0.54$ and 0.90 for x=0.10 and x=1.28 respectively.

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